

Process for Calculating SPARKS-like Life Table in report ZR-047: Fecundity and Mortality

Version 1.0
October 28, 2005

A. Definitions:

The analysis window is the filter used to define the time/geographic period during which data are included in the analysis. It must be defined as a time period, but additional filters may be used as well, such as geographic restrictions (all animals in this set of zoos, or this region) as well as other restrictions (e.g., animals that were hand-reared). Need to have adequate filters to define this risk window. One of the filters needs to be user entered individual numbers (select specific individuals to analyse – or import a file of those individuals)

Population is defined as the set of animals that during any period of their life have entered the analysis window.

A. Tally the Data

Create two separate arrays, one for males, one for females with the following column headers for each (see table 1 in “Life Table SPARKS.xls”).

- AGE CLASS (0 = 0-1, 1 = 1-2, 2 = 2-3, etc, in years). Populate this with numbers 0 to maximum age in years any animal in population experienced. When the tallies are finished, the life-table can be truncated two or so age classes older than the oldest age class with animals at Risk.
- RISK Q_x (number of animal at risk of dying during age class x)
- DEATHS (number of deaths)
- RISK M_x (number of animals at risk of reproducing during age class x)
- BIRTHS (# offspring produced)

Create the following variables to track numbers:

- UnkAge = number of animals with unknown birth dates
- UnkSire = number of sires that are not registered in ZIMS or are listed as Unknown or MULT
- UnkAgeSire = number of sires who have unknown birth dates
- UnkDam – number of dams that are not registered in ZIMS or are listed as Unknown or MULT
- UnkAgeDam = number of dams who have unknown birth dates
- NParKnown = number of parents who have known birth dates
- NNeonateBorn = number of births plus number of animals entering population younger or equal to 30 days of age.
- NNeonateDied = number of deaths to animals younger or equal to 30 days of age

Calculate all ages as fractional years. If age is 568 days, then its age in fractional years is $568/365.25 = 1.556$ years (to three decimal places).

For each animal in the population:

- If animal has unknown age, increment UnknownAge by 1 and skip to next individual

* **Calculating survival rates**

- if it is a female, use female array
- if it is a male, use male array
- if it is neither of the above, treat it as half-male, half-female and when incrementing the arrays below, increment by one-half of the stated value in both the male and female tables.
- if it was alive at the start of the analysis window calculate its age in fractional years when it entered the window. Call this AgeIn
- if it was born in the window, AgeIn = 0.0
- calculate its age in fractional years when it left the window. Use the date it died, was exported, or was lost-to-followup to calculate age; if alive at the end of the risk window, use the End date of the risk window to calculate its age. Call this AgeOut.
- Increment the RISK Qx and RISK Mx over the ages that this animal was in the population. However, if the reason the animal left the window was death, then RISK Qx is incremented by 1.0 in its final age class rather than the fractional amount. Examples:
 - AgeIn = 0; AgeOut = 5.7; Exported or LTF: Increment the both Risk Qx and Risk Mx for age classes 0, 1, 2, 3, and 4 by 1; increment age class 5 by 0.7. (If the animal had sex unknown, increment age classes 0 to 4 by 0.5 in both the male and female tables; and increment age class 5
 - AgeIn = 1.67; AgeOut = 2.52; Death: Increment both RiskQx and RiskMx by 0.33 for age class 1. Increment RiskMx by 0.52 for age class 2. Increment RiskQx by 1.0 for age class 2.
 - AgeIn = 5.67; AgeOut = 5.80; Death: Increment RiskQx by 1.0 for age class 5; Increment RiskMx by $0.80 - 0.67 = 0.13$ for age class 5
- If the animal died, increment DEATH by 1.0 for that age.

* **Fecundity**

- if the animal was born in the analysis window:
 - calculate the age if its dam and sire when the animal was born
 - Increment BIRTHS in the female table for the appropriate dam age by 0.5
 - Increment BIRTHS in the male table for the appropriate sire age by 0.5
- if age of dam and/or sire unknown, tally this by incrementing UnkAgeDam by 0.5 and/or UnkAgeSire by 0.5.
- if the dam and/or sire is unknown, increment UnkSire and/or UnkDam by 1.

Next animal

B. Producing the SPARKS Life Table (Table 2)

Create a table with the following headers:

AgeClass:	Annual age classes
Male Mx:	Average number of offspring sired by males in age class x
Male Risk Mx:	Number of males at risk of siring offspring in age class x
Male Qx:	Probability of a male dying during age class x
Male Risk Qx:	Number of males at risk of dying during age class x
Male Px	Probability of a male surviving through age class x (Px = 1-Qx)
Male Lx	Probability of surviving from birth to beginning of age class x
Male Ex	Average # of future years of life for animal at beginning of age class x
Male Vx	Average # of future offspring for animal at beginning of age class x

Repeat these for females...

Using the data from Table 1, for each age class and each sex;

- $M_x = (\# \text{ BIRTHS age class } x) / \text{Risk}M_x$
- $Q_x = (\# \text{ DEATHS age class } x) / \text{Risk}Q_x$
- $P_x = 1 - Q_x$
- $L_x = 1.0$ for age class 0 ($L_0 = 1.0$)
- $L_x = L_{(x-1)} * P_{(x-1)}$
- life-expectancy (E_x):

$$E_x = \frac{\sum_{y=x}^{\max \text{ age}} L_y}{L_x}$$

30 day mortality: $N\text{NeonateDied}/N\text{NeonateBorn}$

C. Adding Reproduction From Unknown Aged Or Unknown Parents (Calculating M'x)

Births to unknown aged or unknown parents need to be included in the life-table or reproduction is underestimated. These births have been tallied in the UnkSire and UnkDam variables above.

Our practice has been to add these births to the distribution of Mx values in proportion to the Mx values that have been observed:

For the male life table:

Let $BIRTHS'_x$ = number of births to age x with unassigned births proportionally added.
Then

$$BIRTHS'_x = BIRTH_x \left[1 + \frac{UnkAgeSire}{N_m} \right]$$

Where N_m = sum of all $BIRTHS_x$ for males.

Then let $M'_x = BIRTHS'_x / RISKM_x$.

For the female life table:

Use the same approach as was done for the males, but replace UnkAgeSire in the above formula with UnkAgeDam.

D. Smoothing Data

We typically have small sample sizes. In the past we have applied smoothing procedures that recalculate values to give a smoother picture of trends.

The User should be given the opportunity to smooth the data multiple times if so desired. Typically, the original data are presented and the user given the option to smooth the data. If the option is selected, M_x and Q_x values are smoothed and the report or graph is presented again. The user then has the option to smooth again. There is no limit to the number of times the user smooths the data. The user should be able to reset the data to the original values at any time.

Each time the data are smoothed, all summary statistics are recalculated.

The smoothing process is as follows:

a. For Q_x : Let \tilde{Q}_x = smoothed Q_x values

$\tilde{Q}_0 = Q_0$ {first smoothed value uses the observed mortality for age class 0}

The smoothed value for the second age class (\tilde{Q}_1) is the average of the Q_x for the second and third observed values. ($\tilde{Q}_1 = [Q_1 + Q_2]/2$).

All other smoothed values use running means of three: smoothed value for Q_{t-1} = $[Q_{t-1} + Q_t + Q_{t+1}] / 3$.

For the second pass (if the user so desired, the process is repeated so that the second set of smoothed values ($\tilde{\tilde{Q}}_x$) smooths the first set (\tilde{Q}_x).

b. Smoothing M_x

The same process is used for smoothing the M_x values as is used for the Q_x .

These smoothing methods are described in Tukey, J. W. (1977). *Exploratory Data Analysis*, Reading Massachusetts: Addison-Wesley.

E. Dealing with dirty data

Date estimates:

For any dates that are estimated, the life-table calculations should use the date that is the mid-point of a specified range, or the estimated date if it's a date with a date field entered with a +/- field. [Note: Future versions of ZIMS will export raw(er) data and actual dates will be randomly drawn from these ranges]

Unknown Parents and Unknown Aged Parents:

When parents are unknown, or their ages are unknown, their contribution to fecundity can not be calculated since the age of the parents (x) is indeterminable. The variables UnkAgeDam and UnkAgeSire are incremented, and used as described above to calculate M'_x . Note that UnkAge... is incremented when ages of parents are unknown – thus are also incremented when parents are unknown as their ages are also unknown.

Multiple parents:

The SPARKS method treats Multiple parents as unknowns and increments UNKDAM and/or UNKSIRE.

Changing sex:

The SPARKS method uses the most recently entered sex for a specimen.

F. Graphics

The first report seen by the user should be the data table. The user should then be given the option to view graphics. If no smoothing has been done, the graphics should show the male and female values for the variables selected. If smoothing has been done, the user should be given the option of seeing and one of the following:

- the male and female values of the raw data
- the male and female data of the smoothed data
- the male data raw and smoothed
- the female data raw and smoothed

G. Summary Statistics from Life-Tables

The following summary statistics need to be calculated:

- life-table intrinsic rate of increase (r) :

The sum is over all x ages in the table, r is the value that makes this sum = 1.0000. Should solve r to the nearest 3 sig. digits. The solution of r can only be found through iteration.

$$\sum L'_x M'_x e^{-rx} = 1.0$$

Use this value of r in the formulas below.

- life-table lambda (λ):

$$\lambda = e^r$$

- Net reproductive rate (R_0):

$$R_0 = \sum L'_x M'_x$$

- generation length (T):

$$T = \frac{\sum xL'_x M'_x e^{-rx}}{\sum L'_x M'_x e^{-rx}}$$

- Reproductive value (V_x). This is calculated for each age class, but its listed here because r is used in its formula:

$$V_x = \left(\frac{e^{rx}}{L'_x} \right) \sum_{y=x}^{\max age} L'_y M'_y e^{-ry}$$

H. Business rules

- This method only applies to individuals, not groups